Al-Khwarizmi Contributions

Al-Khwarizmi used a number of different processes to solve the equations. He named the different process al-jabar and al-maqubala.

> **Al-jabar** means "adding the same positive quantity to both sides of an equation so as to remove negative terms."[i]
>
> **Al-maquabala** means "the same power appears on both sides, the smaller member on the one side is subtracted from the greater one on the other side"[ii]

Al-Khwarizmi did not make use of any symbols. He called the 'x' shai', which can be translated as thing or root. The x^2 is called mal which can be translated as wealth or property. Al-Khwarizmi listed six basic types of equations which all the terms are positive and there is a least one positive root.

1. Roots equal to numbers.($nx=m$)
2. Mal equal to roots. ($x^2=nx$)
3. Mal equal to numbers. ($x^2=m$)
4. Numbers and mal equal roots ($m+x^2=nx$)
5. Numbers equal roots and mal ($m=nx+x^2$)
6. Mal equals numbers and roots ($x^2=m+nx$)[iii]

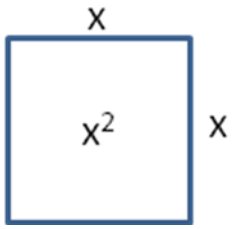
Al-Khwarizmi wrote out his equation in everyday language as can be seen in this example. "Consider the equation$x^2 + 10x = 39$, which he expressed in the form: '*Property and ten things equals thirty-nine'*. His solution reads as follows: *'Take the half of the number of the things, that is five, and multiply it by itself, you obtain twenty-five. Add this to thirty-nine, you get sixty-four. Take the square root, or eight, and subtract from it one half of the number of things, which is five. The result, three, is the thing*.'"[iv]

Al-Khwarizmi also used geometric methods to solve equations. However, Al-Khwarizmi did not know how to represent negative numbers geometrically which is why al-jabar and al-maqubala were used to remove the negative terms of an equation. The attached resource shows how Al-Khwarizmi solved some equations using geometric methods. However, Al-Khwarizmi's geometric methods only gave one positive solution. This was because Al-Khwarizmi was chiefly concerned with using algebra to solve real-life problems.

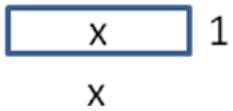
Solving quadratic equations using manipulatives

Today, manipulatives can be used to solve quadratic equations. Different colors are used to represent positive and negative values. The different shapes are listed below.

A large square represented x^2. The dimensions of the large square were ‘x’ by ‘x’.



A rectangle represented ‘x’. The dimensions of the rectangle are ‘x’ by ‘1’.



A small square represented ‘1’ with dimension ‘1’ by ‘1’

There are some conventions to using manipulatives.

1. The quadratic equation should be equal to zero.
2. The ax^2 should be positive.
3. There should be no space between the manipulatives.
4. The x^2 squares go in the top left hand corner while the small squares go in the bottom right hand corner.
5. The manipulatives should form a solid rectangular shape.

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[i] Cooke, 264.

[ii] Kvasz, Ladislav, “The History of Algebra and the Development of the Form of its Language,” *P* Philosophia Mathematica (III) 14 (2006), 292.

[iii] Berggren, 103.

[iv] Kvasz, 292.